The Relationship between Seats and Votes in Two-Party Systems*

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Arrangements for translating votes into legislative seats almost always work to benefit the party winning the largest share of the votes. That the politically rich get richer has infuriated the partisans of minority parties, encouraged those favoring majority parliamentary rule, and, finally, bemused a variety of statisticians and political scientists who have tried to develop parsimonious descriptions and explanations of the inflation of the legislative power of the victorious party.

This paper reports tests of three different models of the relationship between seats and votes against data from 132 elections in six two-party systems. One of these models, the well-known "cube law," meets these tests less well than a rather simple rule of thumb. A more important conclusion from the analysis, however, is that electoral systems differ greatly in both their rate of translation of votes into seats (the "swing ratio") and in their partisan bias. Following the descriptions of the seats-votes relationship are explanations of the observed differences in swing ratio and partisan bias. The political consequences of these differences are assessed over the years for a variety of electoral systems—and it is clear that recent reapportionments in the United States have had dramatic and unexpected effects on the translation of votes into seats. Finally, the measures of swing ratio and partisan bias appear useful for evaluating the consequences of redistricting plans, and might well be used for that purpose by the courts.

The Data and Basic Characteristics and the Relationship

Over a series of elections, the relationship between seats and votes in most two-party systems displays four obvious characteristics (Figure 1 shows the data[1]):

(1) As the party's share of the vote increases, its share of the seats also increases in a fairly regular fashion.

(2) The party that receives a majority of the votes usually receives a majority of parliamentary seats. Such was the case in 93 per cent of the national elections and 53 per cent of the state elections examined here. The points in the upper left and lower right quadrants represent those elections in which the party winning a majority of votes failed to take a majority of seats. New Jersey, like many other states prior to redistricting (and some after redistricting), shows many markedly biased outcomes, with the Democrats often winning fully three-fifths of the votes but less than one-third of the seats.

(3) A party that wins a majority of votes generally wins an even larger majority of seats. This was found in 89 per cent of the 64 national elections in which the party with a majority of votes also won parliamentary control. For the three U.S. states, the comparable figure was 83 per cent. In Rae's study of 117 elections in 20 countries (including multi-party systems), it was 91 per cent of the elections.[2]

(4) In most elections (100 per cent in this series), the winning party receives less than 65 per cent of the votes (although it may receive a much larger share of seats).

Even a casual inspection of the data displayed in Figure 1 indicates that almost any curve with a slope around two or three in the region from 35 to 65 per cent of the vote for a party will fit the relationships rather well. Let us now turn to three models that seek to describe the relationships

* I wish to thank Joseph G. Verbalis, Harold George, Nita Rome, Judith Rowe, and Richard A. Sun for their help in the collection and the analysis of the data. Orley Ashenfelter, Stanley Kelley, Jr., John L. McCarthy, Frederick Mosteller, Walter F. Murphy, Richard E. Quandt, and Dennis F. Thompson provided advice and criticism during the course of the work. Edward Franklin Cox, Gudmund Iversen, and David Mayhew generously gave me access to unpublished materials on congressional elections. The Center for International Studies and the Woodrow Wilson School of Public and International Affairs, Princeton University, provided financial support for the work. Computations were performed at the Princeton University Computer Center, operating under NSF grants GJ-34 and GU-3157. None of these individuals or institutions bears responsibility for the analysis or interpretations of the paper.

[1] The election tabulations were collected from state and national yearbooks. The U.S. congressional returns have been collected together in Donald Stokes and Gudmund Iversen, "National Totals of Votes Cast for Democratic and Republican Candidates for the U.S. House of Representatives, 1866–1960," July, 1962, mimeo, Survey Research Center, University of Michigan. *Congressional Directories* (Washington, D.C.: United States Government Printing Office) were used to update the Stokes-Iversen compilation and also as the source for tabulations requiring election returns in individual congressional districts. All percentages of the vote were computed from the votes received by the two major parties only.

[2] Douglas Rae, *The Political Consequences of Electoral Laws* (New Haven: Yale University Press, 1967), p. 73.

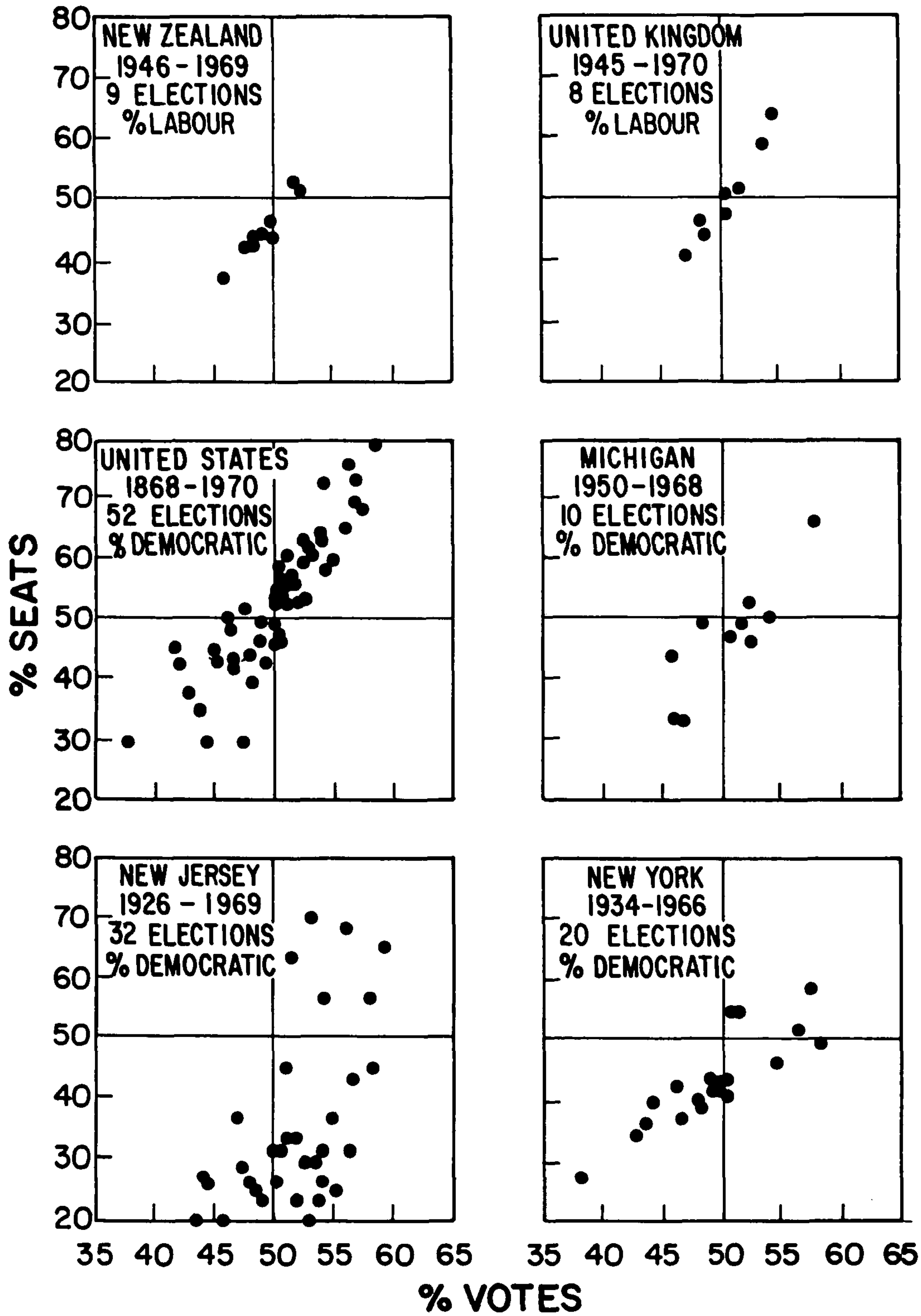


Figure 1. Seats and Votes.

more precisely: a straight line, the "cube law," and a logit fit.

The Linear Fit: Estimates of Bias and Responsiveness

The relationship between seats and votes is described most directly by a simple linear equation:

percentage seats for a particular political party
= b (percentage votes for that party) + c.

The estimate of the slope, $\hat{b}$, measures the percentage change in seats corresponding to a change of one per cent in the votes for a party.[3] Thus $\hat{b}$ estimates the *swing ratio* or the *responsiveness* of the partisan composition of parliamentary bodies to changes in the partisan division of the vote in two party systems. For example, the swing ratio during the last twelve U.S. congressional elections is 1.9, indicating that a net shift of 1.0 per cent in the national vote for a party has typically been associated with a net shift of 1.9 per cent in congressional seats for a party.

In addition, the fitted line provides an estimate of another important parameter of the electoral system: the bias for or against a particular party in the translation of votes into seats. Setting the percentage of seats at 50 per cent and solving for the percentage of votes in the equation of the fitted line tells one the share of the vote that a party typically needs in order to win a majority of seats in the legislative body. The difference between this number and 50 per cent is the *bias* or *party advantage*, as illustrated in Figure 2.[4] For

[3] Robert A. Dahl in *A Preface to Democratic Theory* (Chicago: University of Chicago Press, 1956), pp. 147–149, estimates the linear model for U.S. House and Senate elections.

[4] Somewhat similar notions of bias have been discussed by Butler and MacRae: D. E. Butler, *The Electoral System in Britain Since 1918* (Oxford: Clarendon Press, Second ed., 1963), p. 196; and Duncan MacRae, "Models of Legislative Representation," mimeo, University of Chicago, 1969. This sort of bias can be estimated in four different ways: (1) as described in the text, where the seats-votes line is regarded as a "law"; (2) by regressing *votes on seats* and solving accordingly, (3) by the logit model described below, and (4) by manipulating the distribution of the district vote described below. A brief comparison of the four

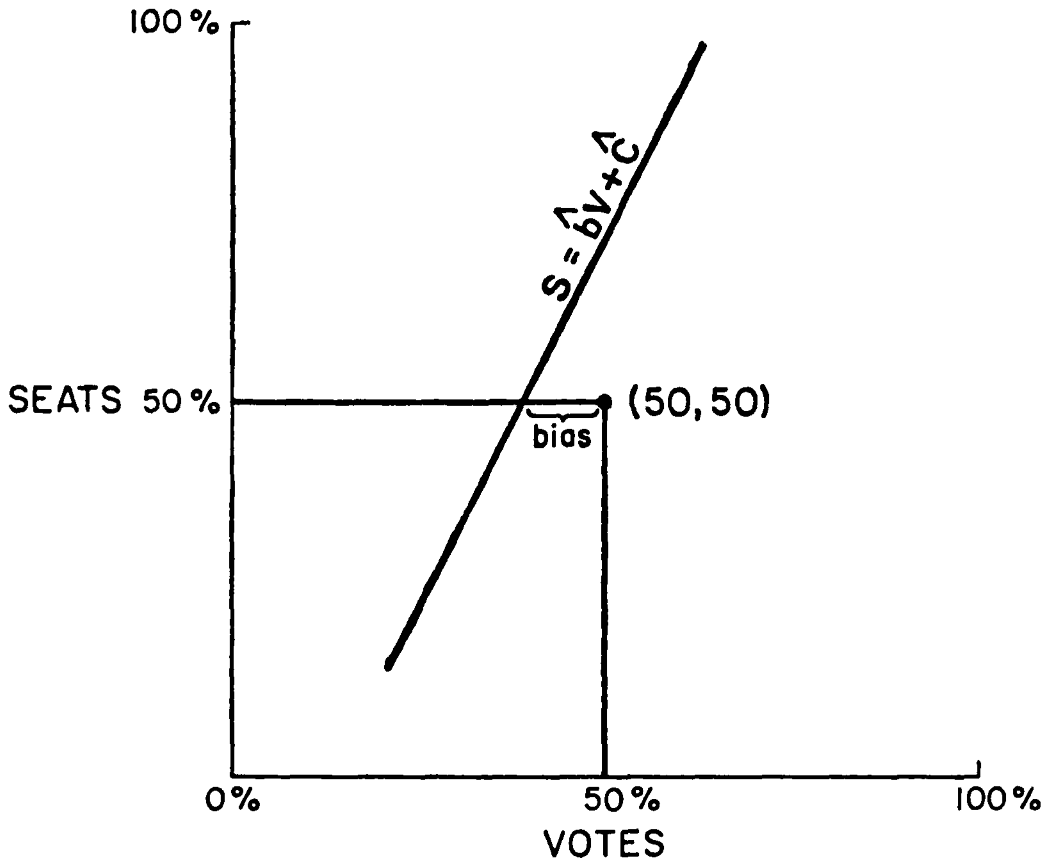


Figure 2. The Fitted Seats-Votes Line.

example, in recent congressional elections, the Democrats have typically needed only about 48 per cent of the national vote in order to win a majority of House seats; thus the bias or party advantage is about 2 per cent. Later we will explain some of the variations in the swing ratio and bias for different electoral systems over the years.

One minor defect of the linear fit is that in general the fitted line will not pass through the end points (0 per cent votes, 0 per cent seats and 100 per cent votes, 100 per cent seats), which are on the seats-votes curve by definition. Although slightly inelegant, this shortcoming is hardly troublesome—especially since parties in two party systems almost never get less than 35 per cent of the vote nor more than 65 per cent of it.

methods revealed small differences in most estimates when the bias was less than 5 per cent and the correlation between seats and votes was fairly high (usually the case); otherwise the estimates diverged. Confidence intervals for the estimated bias, if one believes they are appropriate for these data, can be constructed for the first three methods. For the complex case of method 1, see Albert H. Bowker and Gerald J. Lieberman, *Engineering Statistics* (Englewood Cliffs, New Jersey: Prentice-Hall, 1959), p. 253; for methods 2 and 3, the usual confidence intervals in regression apply. Still another interesting bias is suggested by the question: What proportion of the seats does a party receive when it wins 50% of the vote? For the fitted line, this bias is, by the geometry of the situation, the product of the slope and the bias described in Figure 2.

The clear advantage of the linear fit is that it yields two politically meaningful numbers, the swing ratio and the bias, that can be compared over time and electoral systems.

Table 1 records the fitted lines for a variety of elections. The swing ratios and the biases show considerable variation both between electoral systems and within some systems over time. Among the countries, Great Britain has the greatest swing ratio at 2.8. In the United States, the swing ratio has been about two, although, as we shall see later, there is evidence that in the last few elections, the swing ratio has decreased considerably. The U.K. electoral system shows little bias; in the U.S., a persistent bias has favored the Democratic party—partially the result of that party's victories in small congressional districts and in districts with low turnouts. In Michigan, New Jersey, and New York, there have been large biases favoring the Republicans and a great deal of variation in swing ratios. The relationship between votes and seats is weaker for the three states than for the three countries; in fact, in the states during some time periods there was virtually no correlation between the share of seats that a party won in the legislature and the share of votes it had received at the polls! In more recent elections, however, there was a fairly strong relationship between seats and votes in all three states—probably the result of new rules and practices for districting.

Table 1. Linear Fit for the Relationship Between Seats and Votes

	$\hat{b}$ Swing ratio and (standard error)	r^2	Percentage votes required to give the indicated party a majority of seats in the legislature	Advantaged party and amount of advantage
Great Britain, 1945–1970	2.83 (0.29)	.94	50.2% Labour	Conservatives, 0.2%
New Zealand, 1946–1969	2.27 (0.27)	.91	51.4% Labour	National, 1.4%
United States, 1868–1970	2.39 (0.21)	.71	49.1% Democrats	Democrats, 0.9%
United States, 1900–1970	2.09 (0.14)	.87	48.0% Democrats	Democrats, 2.0%
United States, 1948–1970	1.93 (0.29)	.81	48.8% Democrats	Democrats, 1.2%
Michigan, 1950–1968	2.06 (0.41)	.76	52.1% Democrats	Republicans, 2.1%
New Jersey, 1926–1947	2.10 (0.44)	.53	61.3% Democrats	Republicans, 11.3%
New Jersey, 1947–1969	3.65 (0.89)	.63	52.0% Democrats	Republicans, 2.0%
New York, 1934–1966	1.28 (0.19)	.73	54.3% Democrats	Republicans, 4.3%

The fitted straight line estimates two measures of the performance of the electoral system—the responsiveness and the bias—from the outcomes of several previous elections. Confusion between the effects of swing ratio and bias sometimes occurs when only one election is considered at a time; thus one noted student of apportionment writes:

> In New Jersey in 1966 the Democrats gained a 9–6 edge in congressional seats, despite a Republican plurality in the popular vote. By contrast, for the state legislature in New Jersey, using a mixture of new single- and multi-member districts, a comfortable but not overwhelming Republican plurality in popular votes in 1967 produced a sweep of two-thirds of the seats in each house. Many other examples could be given of these gross imbalances between popular votes received by a party and the number of legislators elected from that party, both before and after one man, one vote, revisions.[5]

Although there is a "gross imbalance" between votes and seats in these two cases, they differ both with respect to the causes of the imbalance and the appropriate normative evaluation of it. In the case of New Jersey congressional elections from 1964 to 1970, there was a bias of 7 per cent; the swing ratio was 1.6. In elections to the New Jersey Assembly from 1965 to 1969, there was a bias of less than one per cent but a swing ratio of 3.6. Thus, the imbalance between seats and votes in congressional elections was due to bias in the districting arrangements; in elections for the Assembly, in contrast, it was due to a steep swing ratio (a consequence of multimember districts) in what has recently become a relatively unbiased system. The two situations, then, differ: electoral systems biased toward a particular party are hardly defensible, at least in relatively democratic systems; very different swing ratios can be justified, however, depending on the objectives sought in constructing an electoral system.

Before examining the consequences and causes of differences in swing ratios and biases in detail, let us consider some alternatives to the straight line—the famous "cube law," and a logit model.

Disposing of the Cube Law

One well-known description of the relationship between votes and seats in two party systems is the "cube law," which has even become part of the political folklore of Great Britain. Although the law was originally proposed by Edgeworth and Smith near the turn of the century and then frequently discussed by David Butler in the Nuffield series of election studies, the source most widely cited in support of the cubic relationship between seats and votes is the paper by Kendall and Stuart.[6] The most economical statement of the law is that the cube of the vote odds equals the seat odds, where the vote odds are the ratio of the share of the votes received by one party divided by the share of the votes received by the competing party. For example, if both parties win 50 per cent of the votes, then the odds are one to one. Figure 3 shows the line traced out by the cube law.

Since Kendall and Stuart wrote, quite a number of papers have touched upon the law and, in the last few years, the law has enjoyed a certain vogue and has been fitted to electoral outcomes in England, the United States, New Zealand, and, in a modified form, in Canada.[7] With one or two exceptions, discussions of the law are quite sympathetic, suggesting that it is a useful and accurate description of electoral realities. Most studies consider no more than a few data points and conclude that the law fits rather well—although the quality of fit is usually assessed informally and no alternative fits are tried. Let us consider a direct test of the predictions of the cube law. The law is

$$\frac{S}{1-S} = \left(\frac{V}{1-V}\right)^3$$

The ratio of shares of seats and votes won by the two parties represents the odds that a party will win a seat or a vote. Taking logarithms yields

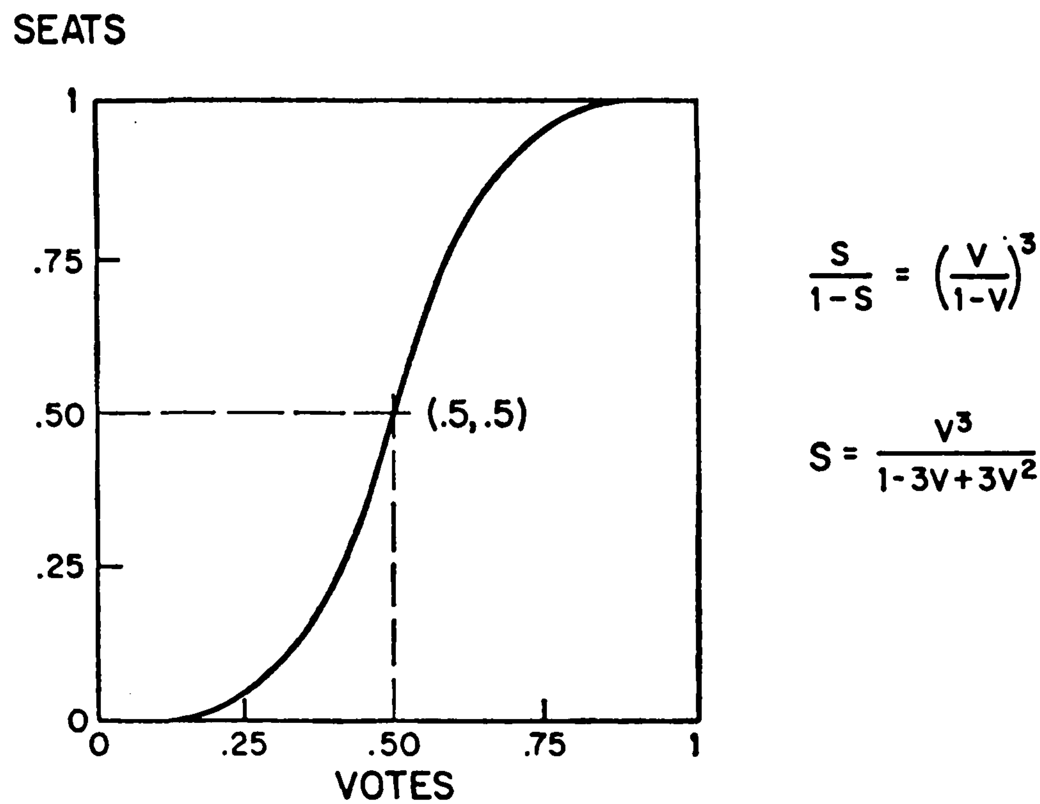
$$\log_e \frac{S}{1-S} = 3 \log_e \frac{V}{1-V},$$

and therefore in the regression of log-odds on seats against log-odds on votes,

[5] Robert G. Dixon, Jr., "The Court, The People, and 'One Man, One Vote,'" in *Reapportionment in the 1970s*, ed. Nelson W. Polsby (Berkeley: University of California Press, 1971), p. 13.

[6] M. G. Kendall and A. Stuart, "The Law of Cubic Proportions in Electoral Results," *British Journal of Sociology*, 1 (September, 1950), 183–197; Kendall and Stuart, "La Loi du Cube dans les Elections Britanniques," *Revue Française de Science Politique*, 2 (April-June, 1952), 270–276.

[7] D. E. Butler, *The British General Election of 1951* (London: Macmillan, 1952), pp. 275–276 and the other Nuffield College election studies; James G. March, "Party Legislative Representation as a Function of Election Results," *Public Opinion Quarterly*, 11 (Winter, 1957–58), 521–542; "Electoral Facts," *The Economist*, January 7, 1950, 5–7; Terrence H. Qualter, "Seats and Votes: An Application of the Cube Law to the Canadian Electoral System," *Canadian Journal of Political Science*, 1 (September, 1968), 336–344. Doubts about the cube law are expressed by Samuel J. Eldersveld, "Polling Results and Prediction Techniques in the British General Election of 1950," in James K. Pollock, *British Election Studies* (Ann Arbor: George Wahr Publishing Co., 1951), pp. 75–78; and in Ralph H. Brookes, "Legislative Representation and Party Vote in New Zealand: Reflections on the March Analysis," *Public Opinion Quarterly*, 23 (Summer, 1954), 288–291.



S = proportion of seats for one party
1 − S = proportion of seats for the other party in a two party system
V = proportion of votes for one party
1 − V = proportion of votes for other party

Figure follows James G. March, "Party Representation as a Function of Election Results," *Public Opinion Quarterly*, 11 (Winter, 1957–58), 524.

Figure 3. The Cube Law.

$$\log_e \frac{S}{1-S} = \beta_0 + \beta_1 \log_e \frac{V}{1-V},$$

the cube law makes the simultaneous joint prediction that $\beta_0 = 0$ and $\beta_1 = 3$. Table 2 reports the results of tests of these predictions.

The table indicates that the cube law fits poorly in six of the seven trials. It fits quite well for the last eight elections in Great Britain, but otherwise its predictions are not confirmed. In short, it is not a "law." Since previous studies have not tested the exact joint predictions of the cube law (that is, $\beta_0 = 0$ and $\beta_1 = 3$) or used as extensive a collection of data, these results should be decisive in evaluating the empirical merits of the cube law. Even in the case of Great Britain, the linear model (and the rule of thumb replacing the cube law proposed below) fits just as well as the cube law, is simpler in form, and is politically more informative.

"But," a defender of the cube law might reply at this point, "aren't there some sound theoretical underpinnings of the cube law—and even though it may not fit very well statistically, doesn't the law retain some merits at least as approximate quantitative theory?" First, it is not clear from any of the literature that there actually *is* a theory behind the cube law. If there is, what is it? One recent justification begins with a seemingly plausible logit model based on the notion that district swings are "harder" in legislative districts going from, say, 75 to 80 per cent (from one election to the next) than from 55 to 60 per cent.[8] While such a logit percentage transform seems statistically natural, it is counterfactual. Percentage swings are relatively independent of the starting point and are therefore best assessed in terms of untransformed percentage differences—the so-called "paradox of swing."[9] Other theoretical justifications do not

[8] Henri Thiel, "The Cube Law Revisited," *Journal of the American Statistical Association*, 65 (September, 1970), 1213–1219.

[9] David Butler and Donald Stokes discuss the issue in detail in *Political Change in Britain* (London: Macmillan, 1969), pp. 303–312.

Table 2. Testing the Predictions of the Cube Law (and Simultaneously Estimating the Logit Model)

	$\hat{\beta}_0$	$\hat{\beta}_1$	Standard error of slope	r^2	Does $\beta_0=0$ *and* $\beta_1=3$ as cube law predicts?	Is $\beta_0 \neq 0$; that is, is there a significant bias?
Great Britain	−.02	2.88	.30	.94	yes	no bias
New Zealand	−.12	2.31	.27	.91	no	yes, there is a bias
United States, 1868–1970	.09	2.52	.24	.68	no	yes
United States, 1900–1970	.17	2.20	.15	.86	no	yes
Michigan	−.17	2.19	.43	.76	no	yes
New Jersey	−.77	2.09	.59	.29	no	yes
New York	−.23	1.33	.19	.74	no	yes

Note: The simultaneous joint hypothesis of the cube law was tested with the appropriate F-ratio; even the most relaxed standards ($p=.50$) did not lead to acceptance of the predictions of the cube law (with the exception of Great Britain). The statistical significance of the bias was tested by computing the t-value on the hypothesis that the intercept of the fitted line does not equal zero. A significant bias (at the .05 level or better) was found in all cases except Great Britain. It is perhaps surprising that the results of the tests are so crisp, given the relatively small number of cases. However, since the data points are rather tightly clustered about the fitted curves, the error variance is small and the results show a perhaps unexpected stability.

lead to specific derivations of the swing ratio predicted by the cube law.[10]

The real theoretical defect of the cube law, however, is that it hides important political issues. The law implies that the translation of votes into seats is

(1) unvarying over place and time, and
(2) always "fair," in the sense that the curve traced out by the law passes through the point (50 per cent votes, 50 per cent seats), and the bias is zero.

As we have seen, these implications are not true. The rate of translation of votes into seats differs greatly across political systems, ranging between gains of 1.3 to 3.7 per cent in seats for each 1.0 per cent gain in votes. Also the results in Table 1 indicate that some electoral systems persistently favor a particular party; the votes-seats curve traced out by the data does not inevitably pass close by the point (50 per cent votes, 50 per cent seats).

It might finally be argued that the cube law still has some faint merit because it is a simple, universal summary of how votes are translated into seats. It is not particularly convenient: how many students of politics can recall that the "cube of vote odds predicts the seat odds" and quickly perform the considerable mental arithmetic required to extract the number of seats predicted by the law? The following rule is a more convenient and more accurate summary of the data than the cube law:

> A 1.0 per cent change in votes for a party leads to a change of 2.5 per cent in seats for a party.

In other words, a rough guess of the swing ratio in two-party systems is about 2.5. This rule generates more accurate predictions for our data sets than the cube law. If one wants (perhaps for teaching purposes) a very crude rule of thumb summarizing the history of votes-seats relationship in two-party systems, then the 2.5 rule is preferable to the cube law.

An Alternative Fit: The Logit Model

A third approach to fitting a curve to the relationship between seats and votes is a logit model, which is fully as effective as the linear model and statistically more graceful. Define the odds in favor of a party's winning a seat as $S/(1 - S)$ and the vote odds as $V/(1 - V)$. The logit model is the regression of the logarithm of seat odds against the logarithm of vote odds (a regression used earlier to test the specific predictions of the cube law):

$$\log_e \frac{S}{1-S} = \beta_0 + \beta_1 \log_e \frac{V}{1-V}$$

Since both variables are logged, the estimate of the slope, $\hat{\beta}_1$, is the estimated elasticity of seat odds with respect to vote odds; that is, a change of one per cent in the vote odds is associated with a change of $\hat{\beta}_1$ per cent in seat odds.[11]

The logit model has the advantage over the linear fit of producing a reasonable predicted value for the share of seats for all logically possible

[10] David Sankoff and Koula Mellos, "The Swing Ratio and Game Theory," *American Political Science Review*, 66 (June, 1972), 551–554. For a formal model taking into account some of the findings of the present paper, see Richard E. Quandt, "A Stochastic Model of Elections in Two-Party Systems," unpublished manuscript, Princeton, 1972.

[11] Logit analysis is described in Henri Thiel, *Principles of Econometrics* (New York: John Wiley, 1971), pp. 632–636.

values of the share of votes; the predicted values stay between 0 and 100 per cent seats for any percentage share of votes. As noted earlier, this is only a theoretical virtue since the more extreme values do not occur empirically. The logit model also provides a direct test of the hypothesis that an electoral system is unbiased, since $\beta_0 = 0$ in an unbiased system. As shown in Table 2, there is a statistically significant bias in all cases except Great Britain.

The logit model is statistically more satisfactory than the linear fit; but its coefficients are not as readily interpretable from a political point of view as are those of the linear model. Thus the straight-line fit was chosen as best suited for our purposes.

Some Explanations and Consequences of Differences in Swing Ratios

Why does the swing ratio vary over time and across electoral systems? Such differences depend, of course, on how voters are distributed over electoral districts. If voters are randomly distributed over all districts—that is, if every district is effectively like every other district—then the swing ratio would be very large, since a party that won even slightly more than half the vote would win all the seats. In effect, the whole nation would be a single member district. Now consider the other extreme in the way districts might be constructed: every voter represents herself or himself in parliament in a two-party system. Such an arrangement yields an exact equivalence of votes and seats along with a swing ratio of unity. This example suggests that the more nationally oriented the politics of a county or the more nationalized the forces prevailing in a given election, the greater the swing ratio—other things being equal.

Another line of argument suggests the same conclusion. Suppose a party gains an average of X per cent of the vote from one election to the next. Consider two different ways in which that gain could be distributed over electoral districts:

(1) The party could gain exactly X per cent of the vote in all districts; that is, a *uniform swing* of X per cent in all districts.

(2) The party could gain X per cent averaged over all districts, but, in some districts, the gain would be less than X per cent and, in other districts, the gain would exceed X per cent—yielding an *averaged swing* of X per cent over all the districts.

Under which distribution of swings, the uniform or the averaged, will the party gain the most seats? In general, a uniform swing across all districts yields more seats than the averaged swing. (Of course, if the same amount of swing were properly distributed over the marginal districts, then a party could maximize the number of seats won for a given amount of swing.) For example, compare the results of a uniform swing of 1 per cent in all districts with those of an averaged swing of 1 per cent that reflects a swing of 2 per cent in half the districts and of 0 per cent in the other half. Define D_{49} as the number of districts in which the division of the vote was between 49 and 50 per cent in the previous election (and thus these districts will change hands with a 1 per cent swing favoring the initially minority party). Define D_{48} similarly. The argument now hinges on the assumption that D_{49} is greater than D_{48} (or more generally that the distribution of districts tails off as the division of vote moves away from 50 per cent). With this assumption, which is generally reasonable, a uniform swing of 1 per cent shifts a total of D_{49} seats; the averaged swing of 1 per cent (half at 2 per cent, half at 0 per cent) shifts a total of $1/2\ D_{49} + 1/2\ D_{48}$ seats, which is less than the gain resulting from the uniform shift, D_{49}.

Once again the argument suggests that the more uniform electoral swings are across the nation, the greater will be the swing ratio. Thus, we would expect the swing ratio in Britain to be greater than that in the United States. We would also expect that, in U.S. congressional elections, the swing ratio will be greater in on-year elections with the presidential contest on the ballot than in off-year elections (when national forces are somewhat diminished). These expectations are borne out in both cases.

Stokes, in his comparison of voting for representatives in the U.K. and U.S., has clearly shown that electoral forces are far more nationalized in parliamentary elections than they are in congressional elections.[12] The standard deviation of electoral swings from the national average has been approximately 1.8 in Britain in recent years; the comparable figure for the United States is 5.4. Thus, it is reasonable to find the swing ratio (2.83) to be greater for Britain than for the United States (1.93), as was shown earlier in Table 1.

A comparison between on-year and off-year congressional elections in the United States reveals a similar, although somewhat more fragile, result: the swing ratio is usually greater for blocs of on-year elections than for blocs of off-year elections, reflecting the presumably greater nationalization of congressional elections when they are held simultaneously with presidential elections. It is also clear from Table 3 that there are other sources of variation in the swing ratio in congressional elections (at least over time). One additional source of such variation is the competi-

[12] Donald E. Stokes, "Parties and the Nationalization of Electoral Forces," in *The American Party Systems*, ed. William N. Chambers and Walter Dean Burnham (New York: Oxford University Press, 1968), 182–202.

Table 3. Swing Ratios in On-Year and Off-Year Elections, United States, 1872–1970

	Swing Ratios	
	On-years	Off-years
1952–1970	2.1	1.7
1932–1950	3.3	3.0
1912–1930	1.9	1.1
1892–1910	2.8	3.5
1872–1890	6.8	4.0

Note: Aggregated over time, the differences are significant at the .05 level. The results are not an artifact of the particular time periods chosen; other blocs of elections show the same pattern.

tiveness of districts; the presence of many closely contested districts will lead to higher swing ratios, since small shifts in the vote will change the party control in many districts. Thus, the swing ratio is the product of both the distribution of swings and the distribution of the district party share of the votes upon which the swings are operating. Some control over the distribution of the party share of the vote is provided in the test in Table 3 by breaking the elections up into 20-year blocs. The relationship between the swing ratio and turnover in seats will be examined in greater detail shortly.

The larger swing ratio in on-year elections generally benefits the President's party, since the steeper slope in the votes-seats translation associated with on-year elections turns small gains in votes into relatively large gains in seats. In the next off-year election, however, even if the vote returns to normal, the President's party will typically lose fewer seats than it gained because it is now riding down the less steep seats-votes curve associated with off-year elections.

This observation can be stated in more formal terms. Suppose the President's party increases its share of votes in the on-year election by ΔV, thereby gaining $\beta_1(\Delta V)$ seats (where β_1 is the swing ratio in on-year elections). Let β_2 be the swing ratio in off-year elections. It is observed empirically that $\beta_1 > \beta_2$. In the next off-year election, in order for the out-party to regain those seats lost to the President's party in the previous on-year election, the out-party needs to increase its share of votes up to $\Delta V + E$, yielding $\beta_2(\Delta V + E)$ seats, where E is the excess share of votes needed to overcome the reduced swing ratio in off-year elections. Regaining all the lost seats requires that

$$\beta_1(\Delta V) = \beta_2(\Delta V + E)$$

Solving for E, the excess votes, yields:

$$E = \Delta V \left[\frac{\beta_1}{\beta_2} - 1\right]$$

For 1952–1970, $\beta_1 = 2.0$ and $\beta_2 = 1.7$; thus

$$E = .24(\Delta V)$$

In order to regain its seats lost in the previous on-year election, the out-party needs a shift almost one-fourth greater than the shift in votes which originally won those seats for the President's party. For example, if in an on-year election the President's party gained four percentage points in votes over its previous winnings (and consequently about 8.4 per cent in seats, given the swing ratio), then the out-party would need a vote shift of $1.24 \times 4\% = 5\%$ to regain the lost seats.

Sources of Bias

The party biases computed earlier result from gerrymandering, differential turnout across districts, and the different population sizes of electoral districts. The purpose of gerrymandering is to shift the seats-votes curve and thereby produce a party advantage. A party advantage may also arise when the votes of districts with widely different turnouts or sizes are aggregated, as in the case of many formerly malapportioned state legislatures. Consider, for instance, the total votes and total seats as they are aggregated over districts. For the House of Representatives, each district adds 1/435 to the seats total; but some districts may add a much smaller share to the total votes for each party. If, in the aggregate of all districts, low turnout or small districts are aligned with a particular party, there will be a bias in the seats-votes curve since that party is winning seats with relatively small numbers of votes.

The persistent Democratic advantage in congressional races is partially the consequence of the many low turnout districts in the South which have usually added much to the Democratic seat total, but little to the Democratic vote total. An additional source of Democratic advantage, prior to the equalization of population size of congressional districts, was the tendency for Democrats to come from smaller districts than Republicans, both in and outside the South. For the 1962 congressional elections, Table 4 indicates that Republican candidates tended to win in the larger districts; thus Republican seats were more expensive in terms of votes than Democratic seats.

The relationship between the district population size and party vote has, surprisingly, persisted even after the extensive redistricting of recent years. Although the Republicans now waste somewhat fewer votes in large districts (because of the reduction of variation in district size), there remains, as Table 5 shows, a strong link between oversized districts and Republican dominance.

Variations in swing ratio and bias can be studied much more deeply with data from individual election districts. With disaggregated data, many different seats-votes curves can be fitted for various periods and states in the country. Indeed, for the states with more than a handful of congressional districts, any districting arrangement (proposed or actually realized) can be evaluated for responsiveness and bias.

Why the Swing Ratio Has Declined in Recent Congressional Elections: The Fruits of Redistricting

We now examine changes in the swing ratio in elections for the U.S. House of Representatives. Table 6 shows estimates of swing ratio and bias for congressional elections for the last hundred years. It appears that a shift—in fact, a rather striking shift—in the relationship between seats and votes has taken place in the last decade. The 1966–1970 triplet displays the second lowest swing ratio of the 17 election triplets since 1870. No doubt the recent elections provide a somewhat narrow range of electoral experience; the Democrats won with votes between 50.9 and 54.3 per cent (a range in votes that is the fifth smallest of the 17 triplets). Until the Republicans control Congress or the Democrats win more decisively, the "new" swing ratio and bias will not be well estimated. The bias is a spectacular 7.9 per cent, reflecting the two close votes that yielded the Democrats a substantial party majority in the House. The estimate of the bias for the 1966–1970 election triplet is, however, somewhat more insecure than for previous blocs of elections because the error of the estimated bias is proportional to the reciprocal of the swing ratio—and in this case the swing ratio is moderately small.

An alternative method of determining the seats-votes curve, used by David Butler in studies of recent British elections, provides confirmation of the relatively large bias and small swing ratio. Figure 4 shows a seats-votes curve for the 1968 election constructed by taking the distribution of the vote by congressional district and watching what happens when the vote changes by ± 1 per cent, then ± 2 per cent, and so on in each district. For example, if the Democratic vote in 1968 had increased by 1 per cent across all districts, the Democrats would have gained an additional four seats; if it had decreased by 1 per cent, the party would have lost two seats. Similarly, a uniform Democratic gain of 2 per cent across all districts would be worth a total of nine seats; a loss of 2 per cent would yield a loss of six seats. (Note the low swing ratio implied by these results.) The assumption involved in the construction of this type of seats-votes curve is that there is a uniform and universal swing from the current returns—an assumption somewhat more appropriate to the U.K. than the U.S. Figure 4 shows, in the region of the actual electoral result, a relatively flat slope with a swing ratio around 1.0. And there is a relatively large bias of approximately 3 per cent. Therefore, although lacking in complete statistical security, the estimates of swing ratio and bias in

Table 4. Population Size of Congressional Districts: 1962 Congressional Elections

Population of Districts	Percentage of seats won by Republicans	Number of Districts
More than 500,000	53%	66
400,000–500,000	44%	152
300,000–400,000	40%	151
Less than 300,000	27%	32
District Size	**Percentage of seats won by Republicans**	**Number of Districts**
Outside the South		
Oversized Districts (15% or more above norm of equal population)	61%	59
Undersized Districts (15% or more below norm of equal population)	40%	53
The South		
Oversized Districts	23%	26
Undersized Districts	0%	32

Source: William B. Prendergast, "Memorandum on Congressional Districting," in *Reapportionment*, ed. Glendon Schubert (New York: Charles Scribner's Sons, 1965), p. 202.

Table 5. Population of Congressional Districts by Party Control, 1970

Population of Districts	Percentage of seats won by Republicans	Number of Districts
More than 550,000	58%	57
500,000–550,000	48%	58
450,000–500,000	47%	114
400,000–450,000	38%	133
Less than 400,000	21%	43

Source: Computed from corrected tabulations in Michael Barone, Grant Ujifusa, and Douglas Matthews, *The Almanac of American Politics* (Gambit, 1972).

Table 6. Three Elections at a Time: Estimates of Swing Ratio and Bias

Years of elections	Swing ratio	Standard error of swing ratio	Percentage of votes to elect 50% seats for Democrats	Size of Democratic party advantage
1870–74	6.01	0.99	51.4%	−1.4%
1876–80	1.48	3.21	50.0%	0.0%
1882–86	3.30	2.83	50.8%	−0.8%
1888–92	6.01	1.36	50.9%	−0.9%
1894–98	2.82	0.09	51.7%	−1.7%
1900–04	2.23	1.09	50.1%	−0.1%
1906–10	4.21	1.38	48.8%	1.2%
1912–16	2.39	0.20	48.8%	1.2%
1918–22	1.96	0.08	47.6%	2.4%
1924–28*	−5.75*	0.36*	40.8%*	9.2%*
1930–34	2.28	0.37	45.9%	4.1%
1936–40	2.50	0.52	47.1%	2.9%
1942–46	1.90	0.60	48.1%	1.9%
1948–52	2.82	1.25	49.5%	0.5%
1954–58	2.35	0.83	50.1%	−0.1%
1960–64	1.65	0.84	47.4%	2.6%
1966–70	0.71	0.23	42.1%	7.9%

* The figures estimated for the 1924–1928 election triplet are peculiar because of the extremely narrow range of variation in the share of the vote (42.1%, 41.6%, and 42.8%) during that period. The average range within an election triplet is about 6%.

recent elections certainly seem worth taking seriously.

Compared with all the other performances of the electoral systems examined in this paper, a system with a swing ratio of 0.7 and a bias of 7.9 per cent, describes a set of electoral arrangements that is both quite unresponsive to shifts in the preferences of voters (as expressed in their party votes for their representatives) and, at the same time, badly biased. How did the low value of the swing ratio for 1966–1970 come about? Certainly the Democratic party, after their substantial gain in votes (3.4 per cent) and relatively tiny gain—given the "normal" swing ratio exceeding 2.0—in seats (3.2 per cent) would like to know what happened in 1970. And for Republicans, 1966 and 1968 need explanation: after all, they managed to make national division of the vote very close but in neither year were they able to win even 45 per cent of the House seats.

Table 7. Marginal Seats: House and Senate, 1956–1970

	Proportion of seats in which the margin of victory was less than 5%	
	House	Senate
1956	20%	13%
1958	22%	11%
1966	16%	9%
1968	14%	15%
1970	13%	13%

Source: computed from vote totals in *Congressional Directories*.

The swing ratio indicates the potential for turnover in representation. The smaller the swing ratio, the less responsive the party distribution of seats is to shifts in the preferences of voters. The extreme case is a swing ratio near zero; such a flat seats-votes curve means that the distribution of seats does not change with the distribution of votes. Figure 5 shows the strong relationship between the swing ratio and the turnover in the House of Representatives for election triplets since 1870. Note the steady drift downward over the years in both the swing ratio and the turnover. Since 1948, the swing ratio has shifted from 2.8 to 2.4 to 1.7, and, most recently, to 0.7. Similarly the turnover in the House has declined, reflecting the long-run decrease in the intensity of competition for congressional seats. Table 7 shows the decreasing number of marginal congressional districts since 1956. In the late 1950s, about 21 per cent of House districts were relatively competitive; in 1970, 13 per cent. (Note the unchanged proportion of marginal Senate seats since 1956.) Several students of Congress have documented the situation in greater detail, and there clearly has been a large decline in competition with a concomitant increase in the tenure of incumbents in the House in recent years.[13]

[13] See: Stokes; Nelson W. Polsby, "The Institutionalization of the U.S. House of Representatives," *American Political Science Review*, 62 (March, 1968), 144–168; and, for a detailed discussion of congressional competition which was particularly helpful

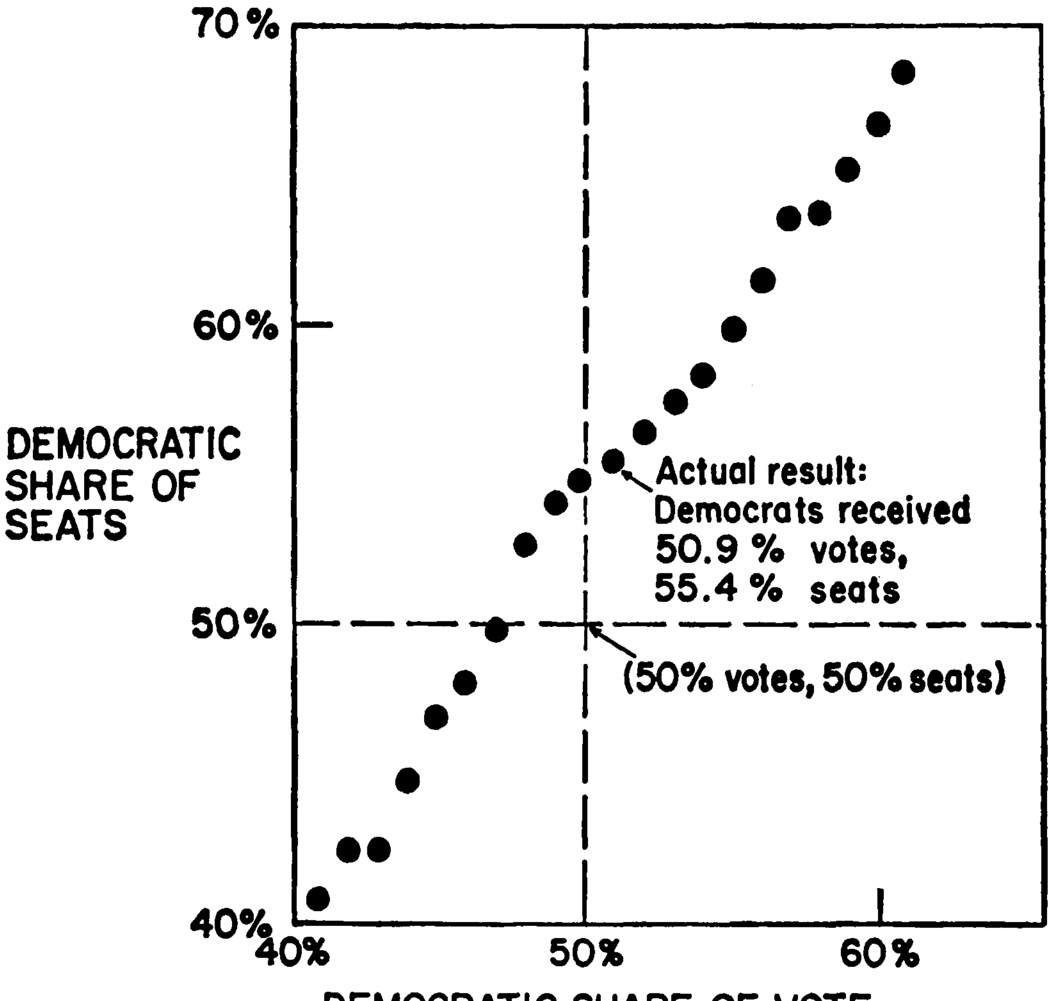


Figure 4. Seats and Votes in 1968.

Our data indicate that a major element in the job security of incumbents is their ability to exert significant control over the drawing of district boundaries; indeed, some recent redistricting laws have been described as the Incumbent Survival Acts of 1972. It is hardly surprising that legislators, like businessmen, collaborate with their nominal adversaries to eliminate dangerous competition. Ironically, reapportionment rulings have given incumbents new opportunities to construct secure districts for themselves, leading to a reduction in turnover that is, in turn, reflected in the sharply reduced swing ratio of the last few elections. One apparent consequence of reapportionment is the remarkable change in the shape of the distribution of congressional votes in recent elections. Prior to 1964, the congressional vote by district was distributed the way everyone expects votes to be distributed: a big clump of relatively competitive districts in the middle, tailing off away from 50% with some peaks at the ends of the distribution for districts without an opposition candidate.

In recent elections the shape of the distribution of the vote by district has changed; Figure 7 shows the movement of district outcomes away from the danger area of 50 per cent in recent years. Note the development of bimodality in the 1968 and 1970 district vote compared to previous years (the left peak contains the Republican safe seats; the right peak contains the Democratic safe seats). Perhaps the best way to see how this pattern de-

in the present analysis, David R. Mayhew, "Congressional Representation: Theory and Practice in Drawing the Districts," in *Reapportionment in the 1970s*, ed. Nelson W. Polsby, pp. 249–290.

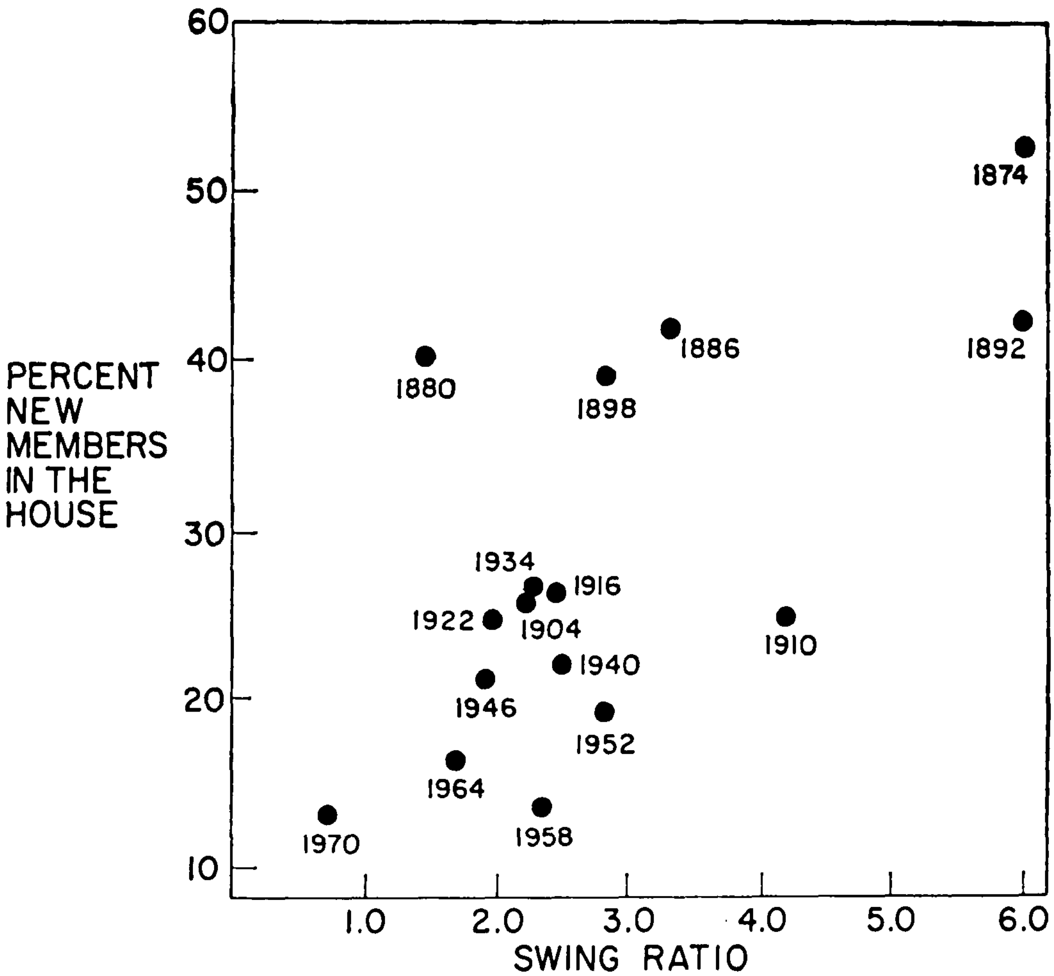


Figure 5. Turnover and Swing Ratio.

Note: The election triplets from Table 6 are used here; the date indicates the last year in the triplet. The percentage of new members is averaged over the three elections.

veloped over time is to array the vote distributions over the years and riffle through them—like an old-time peep show—and watch the middle of the distribution sag and the areas of incumbent safety bulge in the more recent elections.

Many states, through recent reapportionments,

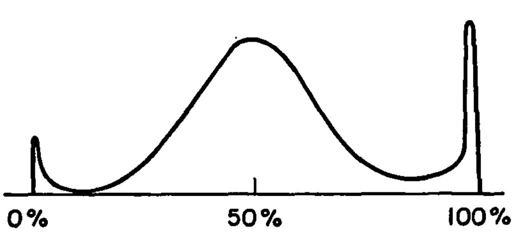


Figure 6. Democratic Share of Vote by Congressional District.

have practically eliminated political competition for congressional seats—even compared to the relatively small proportion of competitive seats in the past. In the 1970 elections in Michigan, for example, not one of the 19 districts was a close contest; the *most* marginal Republican victor won 56% of the vote and the *most* marginal Democrat won fully 70% of the vote in his district. In Illinois, the most closely contested race in all 24 congressional districts in 1970 was a 54-46 division of the vote; in contrast, in 1960, seven districts had closer races than that. The closest 1970 race in Pennsylvania was 55-45; in Ohio, 53-47.

It might be suggested that reapportionment has little to do with changes in competition; after all, the "bipartisan gerrymander," as Mayhew aptly called it, is hardly a recent contrivance growing out of *Baker* v. *Carr*. Perhaps all we are seeing

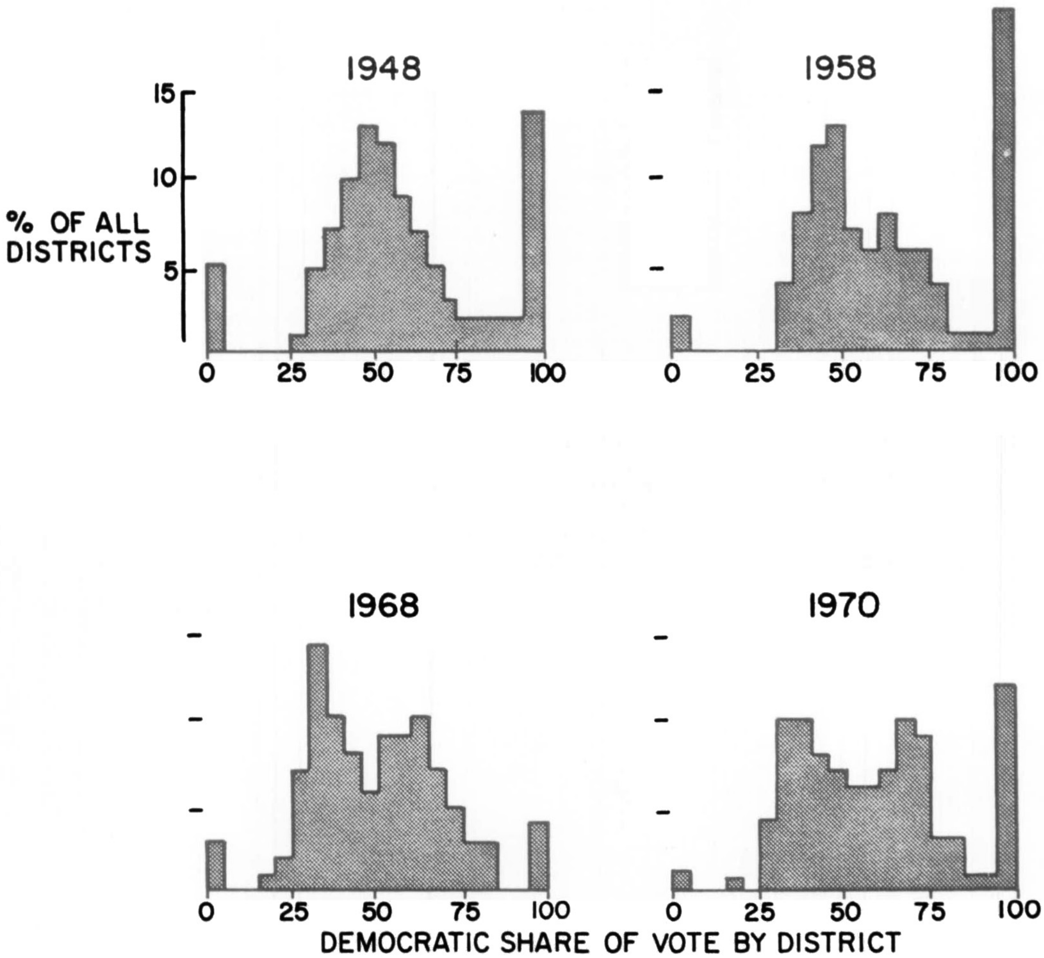


Figure 7. Distribution of Congressional Vote by District: 1948, 1958, 1968, 1970.

here is the continuation of the very long-run decline in competition for congressional seats. The evidence, however, tends rather to indicate that reapportionment has made its own independent contribution to the incumbency of incumbents. The decline in competition is not present in some contests in which the outcome is unaffected by redistricting: Table 7 shows that the marginality of Senate seats has not declined since reapportionment started. The independent contribution of reapportionment to the job security of incumbents can also be seen directly in the elections immediately following reapportionment in a state: there is an immediate decline in the competitiveness of the races in the first election after the new districting.

Concluding Practical Implications: Using the Bias and Swing Ratio to Evaluate the Fairness of Redistricting Plans

Ever since Governor Elbridge Gerry of Massachusetts designed a salamander-shaped district some 160 years ago, politicians have closely attended to the minutiae of political cartography. Control of districting by one political party can mean many additional congressional seats in the larger states or the difference between majority and minority status in a state legislature—often remarkably independent of voters' preferences. And a "bipartisan" redistricting, as we have seen, can virtually end competition for congressional seats within a state. Redistricting should do more than fill the immediate needs of incumbents; such arrangements should pass at least two tests if an electoral system is to be minimally democratic:

(1) The districting should yield an electoral system that is *responsive* to changes in votes. If many citizens shift their votes from one party to another, then the advantaged party should win an increased share of legislative seats.

(2) The districting should be relatively *unbiased* with respect to political party; the electoral system should treat Democrats and Republicans alike.

For a state with more than a few congressional districts and for all state legislatures, it is possible, by constructing seats-votes curves (using the two methods of the previous section), to assess the bias and the responsiveness of any and all redistricting plans—and thereby judge how well a particular plan meets the two tests.

Although reapportionment eliminated some of the cruder distortions in the translations of votes into seats, we now face the problems arising from partisan and bipartisan gerrymandering. The advertised purpose of districting rules such as equal size, compactness, and contiguity was to produce "fair and effective representation." The rules did not achieve these goals. Many redistrictings, although perfectly satisfactory by current legal standards, have produced quite biased and unresponsive electoral systems.

The conventional wisdom views reapportionment as a *district* problem; if we only make enough rules about the construction of *individual* districts, then we will have fair and effective representation. This view is false in logic and in experience. Districting, as every politician knows, is a statewide problem; the lines of one district affect the lines of other districts. Rules specifying the desired virtues of individual districts merely add a few steps to a computer program that sorts through hundreds of statewide districting combinations; such rules cannot prevent the construction of highly biased and unresponsive electoral systems. Redistricting computer programs now seek to maximize incumbency within the constraints placed on individual districts; limits can be placed on such techniques by specifying minimum standards for the responsiveness and bias of the congressional districting in a state. Thus the achievement of an unbiased and responsive electoral system will require action based on the aggregate assessment of districting arrangements.